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COMPUTATIONAL ANALYSES OF MESOCOPIC COMPOSITE REINFORCEMENT DEFORMATION DURING PREFORMING

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SUMMARY: The preforming stage of the LCM composite manufacturing processes leads to fibrous reinforcement deformations which may be very large especially for double curvature shapes. The knowledge of the mesoscopic deformed geometry is important for reinforcement permeability computations. A simulation method for woven composite fabric deformation at mesoscopic scale is presented. . Since yarns are made of thousands of fibers it is not possible to model each of them and an equivalent continuum mechanics model is developed within the hypo-elastic theory.. The associated objective derivative is based on the fiber rotation. X-ray tomography is used to obtain experimental undeformed and deformed 3D geometries of the textile reinforcements
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KEYWORDS: Textile composites, preforming stage, mesoscopic scale, hypo-elasticity, fibrous material, In-plane shear

INTRODUCTION

When a resin flow through a composite reinforcement is analysed, the geometry of this fibrous reinforcement play a major role. It is often complex because of the weaving of the fibres but also because of the deformation due to the preforming stage. These strains are very important when the composite part is double curved. The objective of the present work is to propose an analysis of the deformation of the unit woven cell of the fibrous reinforcement (mesoscopic scale) with a view to simulate the resin flow within this strained unit cell and thus determine the textile permeability

Textile composite reinforcements are made up of fibers. Consequently their mechanical behavior is very specific considering the possible sliding and the interactions between the fibers. When they are formed on double curved shapes, these fabrics are submitted to large strains, in particular large in-plane shear. In the present communication, the textile reinforcement deformation analysis is based on a rate constitutive equation specific to materials made of fibers. The objective

derivative of this law is defined from the fiber rotation, which guarantees a correct stress update during the simulation. This constitutive model is implemented in ABAQUS and can be used in most commercial F.E. software. A second point concerns the boundary conditions that have to render the periodicity at large deformations and, in some cases, the evolution of contacts between neighbouring yarns during the motion.
The numerical analysis of in-plane shear of a unit cell is shown and compared with experimental results.

MECHANICAL BEHAVIOUR MODELLING OF THE YARN

Hypo-Elastic Approach

Rate constitutive equations (or hypo-elastic laws) [1, 2] are very much used in finite element analyses at large strains [3-6]. User subroutines that can be implemented in codes such as ABAQUS to define the mechanical constitutive behaviour are written within this framework. A stress rate $\underline{\underline{\boldsymbol{\sigma}}}^{\nabla}$ is related to the strain rate $\underline{\underline{\mathbf{D}}}$ by a constitutive tensor $\mathbf{C}$. In order to avoid that rigid body rotations affect the stress state, the derivative $\underline{\underline{\boldsymbol{\sigma}}}^{\nabla}$, called objective derivative, is the derivative for an observer who is fixed with respect to the matter. Because this requirement is not uniquely defined there are several objective derivatives. In this work rotational objective derivatives correspond to a rotation tensor $\underline{\underline{\mathbf{Q}}}$ characterizing the rotation of the matter. The rate constitutive equation has the form:

$$\underline{\underline{\boldsymbol{\sigma}}}^{\nabla} = \underline{\underline{\underline{\underline{\mathbf{C}}}}} : \underline{\underline{\mathbf{D}}} \qquad \text{with} \qquad \underline{\underline{\boldsymbol{\sigma}}}^{\nabla} = \underline{\underline{\mathbf{Q}}}.\left(\frac{d}{dt}\left(\underline{\underline{\mathbf{Q}}}^{T}.\underline{\underline{\boldsymbol{\sigma}}}.\underline{\underline{\mathbf{Q}}}\right)\right).\underline{\underline{\mathbf{Q}}}^{T} = \dot{\underline{\underline{\boldsymbol{\sigma}}}} + \underline{\underline{\boldsymbol{\sigma}}}.\underline{\underline{\boldsymbol{\Omega}}} - \underline{\underline{\boldsymbol{\Omega}}}.\underline{\underline{\boldsymbol{\sigma}}} \quad (1)$$

where $\underline{\underline{\boldsymbol{\Omega}}}$ is the spin corresponding to $\underline{\underline{\mathbf{Q}}}$, i. e. $\underline{\underline{\boldsymbol{\Omega}}} = \dot{\underline{\underline{\mathbf{Q}}}}.\underline{\underline{\mathbf{Q}}}^{T}$.

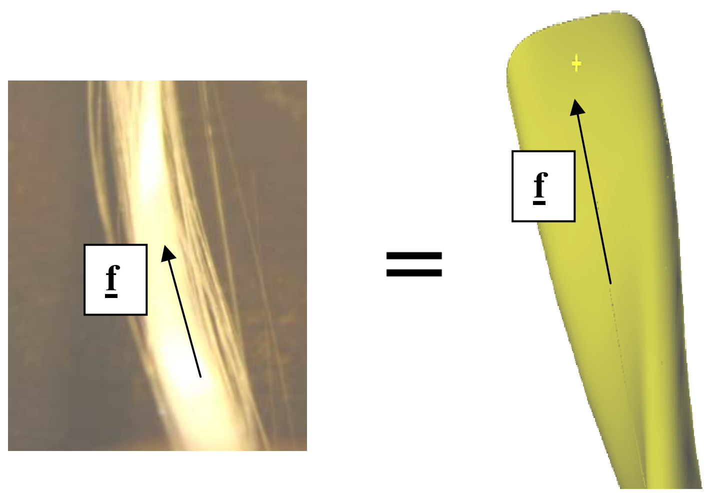
The most usual objective derivatives are those of Green-Naghdi [7, 8] and Jaumann [8-10]. In the case of the derivative of Green-Naghdi the rotation $\underline{\underline{\mathbf{Q}}}$, considered as that of the matter, is the rotation $\underline{\underline{\mathbf{R}}}$ of the polar decomposition, which is derived from the decomposition $\underline{\underline{\mathbf{F}}}=\underline{\underline{\mathbf{R}}}.\underline{\underline{\mathbf{U}}}$ of the gradient tensor. In the case of the derivative of Jaumann, $\underline{\underline{\mathbf{Q}}}$ is the rotation of the corotational spinless frame.

During a finite element analysis the rate constitutive equation is used to update stresses once the displacement field and the corresponding strain field have been computed over the current time increment. Integrating equation (1) over a time increment $\Delta t = t^{n+1} - t^{n}$ leads to the widely used formula of Hughes and Winget [11] for stress update:

$$\begin{aligned} \left[\boldsymbol{\sigma}^{n+1}\right]_{e_i^{n+1}} &= \left[\boldsymbol{\sigma}^{n}\right]_{e_i^{n}} + \left[\mathbf{C}^{n+1/2}\right]_{e_i^{n+1/2}} \left[\Delta\boldsymbol{\varepsilon}\right]_{e_i^{n+1/2}} \\ \text{with } \left[\Delta\boldsymbol{\varepsilon}\right]_{e_i^{n+1/2}} &= \left[\mathbf{C}^{n+1/2}\right]_{e_i^{n+1/2}} \left[\mathbf{D}^{n+1/2}\right]_{e_i^{n+1/2}} \Delta t \end{aligned} \qquad (2)$$

where $\left[\mathbf{S}\right]_{e_i^n}$ denotes the matrix of the components of the tensor $\mathbf{S}$ in the basis $\underline{\mathbf{e}}_i \otimes \underline{\mathbf{e}}_j \otimes \ldots \otimes \underline{\mathbf{e}}_m$ at time t^n. Voigt notation is used: the components of a second order tensor are arranged in a single column matrix. The basis of vectors $\underline{\mathbf{e}}_i^n$ (i = 1, 3) comes from the transportation at time t^n of the initial basis vectors $\mathbf{e}_i^0$ by the rotation $\underline{\mathbf{Q}}$ which defines the objective derivative (1). The frame $\{\underline{\mathbf{e}}_1^n, \underline{\mathbf{e}}_2^n, \underline{\mathbf{e}}_3^n\}$ denoted as $\{\underline{\mathbf{e}}_i^n\}$ is called rotated frame.

Textile materials are made of fibres, which makes their mechanical behaviour very specific. Relative sliding is possible between fibres (see fig. 1a). The yarns are made of thousands or tens of thousands of fibres and it is in general not possible to model each of them. The constitutive model that is introduced in the present paper is a continuum model intended to stand for the specific mechanical behaviour of the fibre bundle (fig. 1b). In this paper a single fibre direction is considered since the mesoscopic scale is the scale of the yarns. The fiber bundle behavior is supposed to be transversely isotropic. The transverse behaviour is thus assumed to be isotropic (though unhomogeneous). This assumption is supported by high resolution tomography observations [12] made on deformed and undeformed reinforcements (Fig. 2).



(a) (b)

Fig. 1. Fibre direction $\underline{\mathbf{f}}_1$ inside a yarn. (a) actual yarn (b) equivalent continuum model.

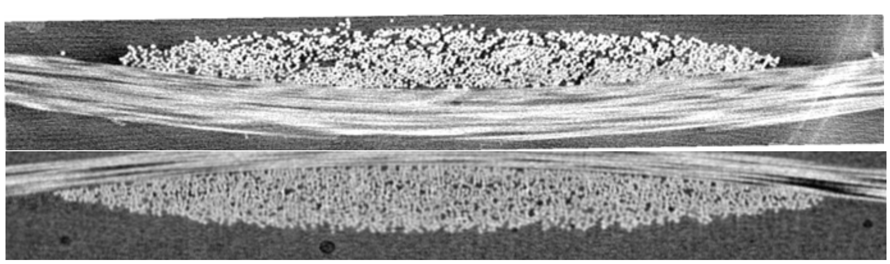


Fig. 2 Tomography reconstructed slices of a glass plain weave. Undeformed state (top) and biaxial tensioned state (bottom) - different scales.

The equivalent continuum behaviour must take into account the fibrous nature of the material. The fibre direction stiffness is much larger than the others. Consequently the constitutive tensor $\mathbf{C}$ is oriented by $\underline{\mathbf{f}}_1$ the unit vector in the direction of the fibre. The direction of the vector $\underline{\mathbf{f}}_1$ is in general not constant in $\{\mathbf{e}_i\}$. Since it is a material direction, the initial fibre direction $\underline{\mathbf{f}}_1^0$ is transformed by $\underline{\underline{\mathbf{F}}}$, the gradient tensor, into $\underline{\mathbf{f}}_1$ while $\{\underline{\mathbf{e}}_i\}$ is rotated by $\underline{\underline{\mathbf{Q}}}$.

To solve this problem the proposed approach [13] consists in using for equation (1) an objective derivative defined from the fibre rotation.

Objective Based on the Fibre Rotation

In the proposed approach, the rate constitutive equation (1) is based on the rotation of the fibre $\underline{\mathbf{f}}_1$:

$$\underline{\underline{\boldsymbol{\sigma}}}^{\nabla_\phi} = \underline{\underline{\underline{\mathbf{C}}}} : \underline{\underline{\mathbf{D}}} \qquad \text{with} \qquad \underline{\underline{\boldsymbol{\sigma}}}^{\nabla_\phi} = \underline{\underline{\boldsymbol{\Phi}}} . \left(\frac{d}{dt} \left(\underline{\underline{\boldsymbol{\Phi}}}^T . \underline{\underline{\boldsymbol{\sigma}}} . \underline{\underline{\boldsymbol{\Phi}}} \right) \right) . \underline{\underline{\boldsymbol{\Phi}}}^T \tag{3}$$

where $\underline{\underline{\boldsymbol{\Phi}}}$ is the rotation of the fibre. It can be shown that this derivative is objective. The stress update (2) becomes:

$$\left[\boldsymbol{\sigma}^{n+1} \right]_{f_i^{n+1}} = \left[\boldsymbol{\sigma}^{n} \right]_{f_i^{n}} + \left[\mathbf{C}^{n+1/2} \right]_{f_i^{n+1/2}} \left[\Delta \boldsymbol{\varepsilon} \right]_{f_i^{n+1/2}} \tag{4}$$

The rotation $\underline{\underline{\boldsymbol{\Phi}}}$ (eq. (3)) from the initial known frame $\{\underline{\mathbf{f}}_i^0\}$ to the current frame $\{\underline{\mathbf{f}}_i\}$ has to be determined. From the transformation gradient $\underline{\underline{\mathbf{F}}}$, the current fibre direction $\underline{\mathbf{f}}_1$ can be determined. Assuming that the initial position of the fibre is $\underline{\mathbf{f}}_1^0$:

$$\underline{\mathbf{f}}_1 = \frac{\underline{\underline{\mathbf{F}}} \cdot \underline{\mathbf{f}}_1^0}{\left\| \underline{\underline{\mathbf{F}}} \cdot \underline{\mathbf{f}}_1^0 \right\|} \tag{5}$$

The other basis vectors $\underline{\mathbf{f}}_2$ and $\underline{\mathbf{f}}_3$ of the orthonormal frame $\{\underline{\mathbf{f}}_i\}$ are obtained from the material transformation of $\underline{\mathbf{f}}_2^0$:

$$\underline{\mathbf{f}}_2 = \frac{\underline{\underline{\mathbf{F}}} . \underline{\mathbf{f}}_2^0 - \left(\underline{\underline{\mathbf{F}}} . \underline{\mathbf{f}}_2^0 . \underline{\mathbf{f}}_1 \right)}{\left\| \underline{\underline{\mathbf{F}}} . \underline{\mathbf{f}}_2^0 - \left(\underline{\underline{\mathbf{F}}} . \underline{\mathbf{f}}_2^0 . \underline{\mathbf{f}}_1 \right) \right\|} \quad \text{and} \quad \underline{\mathbf{f}}_3 = \underline{\mathbf{f}}_1 \times \underline{\mathbf{f}}_2 \tag{6}$$

Then the rotation $\underline{\underline{\boldsymbol{\Phi}}}$ is derived in the following way:

$$\underline{\underline{\boldsymbol{\Phi}}} = \underline{\mathbf{f}}_i \otimes \underline{\mathbf{f}}_i^0 = \left(\underline{\mathbf{f}}_j \right)_i^0 \underline{\mathbf{f}}_i^0 \otimes \underline{\mathbf{f}}_j^0 = \left(\underline{\mathbf{f}}_j . \underline{\mathbf{f}}_i^0 \right) \underline{\mathbf{f}}_i^0 \otimes \underline{\mathbf{f}}_j^0 \tag{7}$$

The main interest of this approach is that the constitutive matrix in equation (4) appears in the frame of the fibre and consequently it is directly in its specific form corresponding to the textile material under consideration. This constitutive matrix written in the fibre frame can be assumed constant in some cases. Generally it is not; the transverse behaviour of a fibrous yarn is depending on the strain state.

When using a material user subroutine in a code such as ABAQUS, the strain increment $\Delta\boldsymbol{\varepsilon}$ is given at Gauss points in a frame which is not $\{\underline{\mathbf{f}}_i\}$ but a standard frame. In the case of

ABAQUS/Explicit it is Green-Naghdi's frame $\{\underline{\mathbf{e}}_i\}$ ($\underline{\mathbf{e}}_i = \underline{\underline{\mathbf{R}}}.\ \underline{\mathbf{e}}_i^0$). To use equation (4) it is necessary to calculate $\left[\Delta\boldsymbol{\varepsilon}\right]_{f_i}$ by a change of basis corresponding to the rotation $\underline{\underline{\mathbf{\Phi}}}.\underline{\underline{\mathbf{R}}}^T$. In the same way, when the stress update is performed with equation (4) it is necessary to return the stress components at t^{n+1} in the code's work frame using an inverse change of basis (corresponding to $\underline{\underline{\mathbf{R}}}.\underline{\underline{\mathbf{\Phi}}}^T$).

Transverse Mechanical Behaviour

Thanks to the use of the fiber frame, the constitutive matrix components along the fibre direction and the transverse ones can be distinguished. The fibre direction modulus is obtained by a tensile test on a yarn. It is considered as constant. The transverse modulus (remember that the transverse behaviour is assumed to be isotropic) is related to the longitudinal and transverse strains. If the yarn undergoes longitudinal tension, it becomes transversely much stiffer. It is also very little stiff transversally when transverse compression is low and it becomes stiffer as compression increases. The following form is used for the transverse modulus E_T:

$$E_T(c, \varepsilon_{11}) = E_0 + k|\varepsilon_{11}|c^2$$

where c is a measure of the transverse compaction namely the local cross section area variation and ε_{11} is the longitudinal strain. The coefficient values E_0 and k have been identified by an inverse method from equi-biaxial tension tests which lead to a significant compaction of the yarn [14]. It was shown in [14] that in order to have a continuum with a yarn type behaviour, i.e. null bending stiffness (or very low), as it is the case for a bundle of several thousands of fibres because of relative sliding of fibres, transverse shear moduli have to be null or very low. Poisson ratios are supposed to be null. The values of the material properties used for the glass plain weave of this study are listed in Table 1.

Table 1 Material parameters

Longitudinal Young modulus E1	35400 MPa
Transverse Young modulus	$0.2 + 8.10^4\|\varepsilon_{11}\|c^2$ MPa
Poisson ratios	0
Shear moduli	20 MPa

MESOSCOPIC SIMULATION OF THE SHEAR OF A GLASS PLAIN WEAVE UNIT CELL

The hypo-elastic material model based on the fibre rotation, introduced in section 2, is used to simulate the in-plane shear of a woven unit cell. There are two types of objectives for such a simulation. Firstly the macroscopic shear mechanical behaviour of the reinforcement can be determined, which is difficult experimentally. The picture frame and bias tests that are used to this aim are delicate. The mesoscopic simulation can also be performed at the design stage of the reinforcement. Secondly, it provides local mesoscopic results such as yarn deformation and shape. These results are very important to perform damage prediction analyses or to determine permeabilities of the reinforcement.

The fabric studied in this paper is a glass plain weave of which the specifications are given in Table 2. The geometric model is established in order to insure its consistency, i.e. there are neither yarn penetrations nor unexpected voids [15]. The mesh of the yarns is mapped, which allows easily defining the initial fibre direction. Next, the choice of the unit cell and boundary conditions has to render the periodicity of the reinforcement. To this end the displacement field is split into a macroscopic average part and a local periodic part, which easily leads to defining kinematical boundary conditions for each material point of the boundary. More details about other requirements can be found in [16].

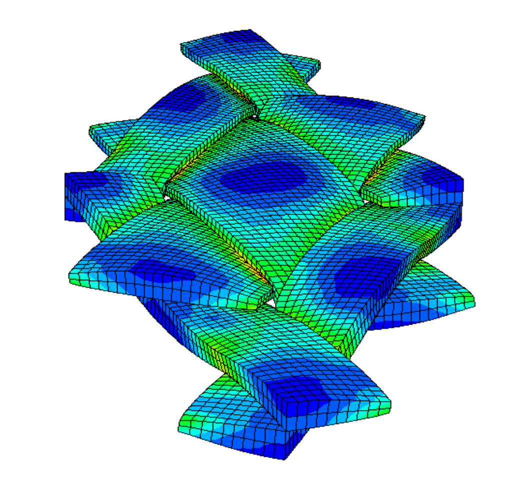
The material parameters (see Table 1) are determined from a tensile test on a single yarn for the Young modulus and from an inverse method with an equi-biaxial tension test for the other parameters. The latter method is interesting to determine the transverse behaviour because it features significant transverse crushing of warp yarns over weft yarns. At last, the friction coefficient it set to 0.2, which is a usual value for friction between carbon fibres.

Table 2 Balanced glass plain weave specifications

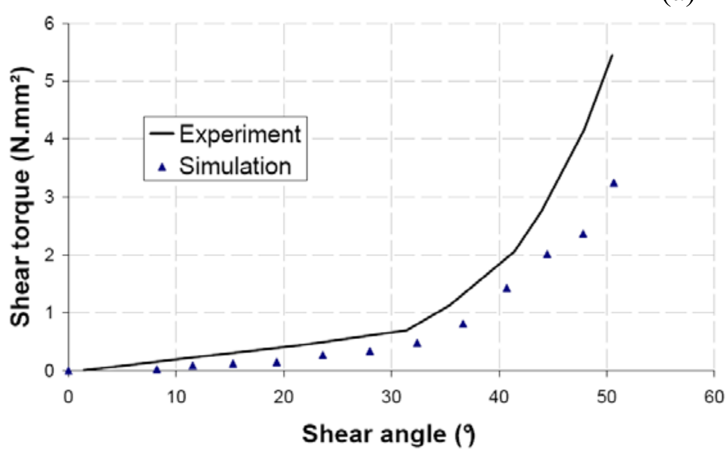
Weaving	Plain weave
Yarn width (mm)	Warp: 3,2 Weft: 3,1
Densities (Yarn/mm)	Warp: 0,251 Weft: 0,248
Crimp (%)	Warp: 0,5 Weft: 0,54
Surface weight (g/m²)	600

Fig. 3b shows the shear curve extracted from the computations compared with the one of an experimental picture frame test. From 35° the stiffness increases significantly with the shear angle because of the yarn locking within the woven cell. From this transition, the shear stiffness is related to lateral crushing of the yarns due to the square geometry turning into a rhomboid one. The agreement is good, keeping in mind that spurious tensions are very difficult to avoid in the picture frame test and tend to slightly overestimate the shear curve at large shear angles. This is the main reason why results may be different from one test to another.

The deformed cell for a shear angle of 53° is shown in Fig. 3a. The local compaction is plotted and this value can locally reach 39%. Though this deformed geometry doesn't suffer any major distortion or defects, it can not be evaluated further at the moment. It is planed to use tomography as an observation tool of deformed woven reinforcements in order to compare the obtained images with the simulated shape.



(a)



(b)

Fig. 3 (a) Deformed unit cell with contours of local compaction;
(b) Shear curve, simulation vs experiment.

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CONCLUSION

The deformation of the fiber reinforcement during preforming is a critical information for flow simulations in order to determine the permeability of the reinforcement [17]. A method for the analysis of this mesoscopic deformation of woven reinforcements was presented. A very important aspect is the yarn constitutive model which is based on a specific transversely isotropic hypo-elastic model developed for large strain analysis of textile composite reinforcements. It is based on an objective derivative defined from the fibre rotation. The approach is simple and can be implemented in any commercial F.E. software. Nevertheless it must be underlined that its efficiency will depend on the quality of the identification of the constitutive matrix. Especially the transverse moduli mainly depend on the fibre compaction (i.e. on strains) and their values are very important for the accuracy of the simulation. The results presented in the case of in plane shear are good compared to the experiments. However, further investigations are needed to evaluate the deformed geometry. This is the point of the work in progress using tomography as an observation tool [12].

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